Anal. Bioanal. Electrochem., Vol. 6, No. 6, 2014, 713-732

Analytical & Bioanalytical Electrochemistry

www.abechem.com

Full Paper

Analytical Expression of Concentration of Substrate in Biofilm Reactor using Adomian Decomposition Method

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Received: 7 November 2014 / Received in Revised form: 26 November 2014 / Accepted: 7 December 2014/ Published online: 31 December 2014

Abstract- Mathematical and kinetic modelings of biofilm reactors were discussed. The model contains a non-linear term related to Monod and Haldane kinetics. In this paper, a simple approximate analytical expression of substrate concentration profiles is derived in terms of all dimensionless parameters. These analytical results were found to be in good agreement with numerical solution by using the Matlab program. Moreover, in we employ modified Adomian decomposition method (ADM) here to solve the second order nonlinear differential equation. This work presents the approximate analytical solution for non-linear equations in a fixed bed biofilm reactor. This analytical result could be very helpful for analyse and to optimize the parameter.

Keywords- Mathematical modeling, Kinetics, Nonlinear equations, Biofilm reactor, Biofuel

1. INTRODUCTION

Biomass, including forestry and agricultural residues, industrial, human and animal wastes, is one of the most important renewable energy sources in the world. Biomass is the term used to describe the renewable-energy sources that come from plants and animals. Biomass is converted into energy by burning it, fermenting it (turning it into alcohol), letting it decay or using chemicals to convert it into a gas or liquid. Recently, trash has been burned to generate electricity. It is interesting to note that, despite the world's dependence on fossil fuels, biomass is the number one energy source used throughout the world [1-3].

Anaerobic digestion is a process by which a complex mixture of symbiotic microorganisms transforms organic materials under oxygen-free conditions into biogas and nutrients. In practice, microbial anaerobic conversion to methane is a process for both effective waste treatment and sustainable energy production. In waste treatment, this process can provide a source of energy while reducing the pollution and odor potential of the substrate. Unlike fossil fuels, use of renewable methane represent a closed carbon cycle and thus does not contribute to increases in the atmospheric concentration of carbon dioxide [4-6].

Methane is important for electrical generation by burning it as a fuel in a gas turbine or steam boiler. Compared to other hydrocarbon fuels, burning methane produces less carbon dioxide for each unit of heat released. The biological activity is confirmed by measuring the gas evolution (CH_4+CO_2), which appears as active bubbles displacing the water in the inverted level jar. Biofilm reactors are a practical alternative for the control and mitigation of these emissions. Biofilm reactors are low cost, energy efficient and are simple to construct and operate [7-8]. The most popular of these is the Monod kinetics, but other models such as the Contois model and the Haldane models are also used sometimes. One has to carefully select a model, which best describes the system under consideration; but, whatever be the kinetic model, a successful modeling of the bioreactor requires an accurate knowledge of the kinetic parameters [9].

Ant colony optimization is a technique for optimization that was introduced in the early 1990's. The inspiring source of ant colony optimization is the foraging behavior of real ant colonies. This behavior is exploited in artificial ant colonies for the search of approximate solutions to discrete optimization problems, to continuous optimization problems, and to important problems in telecommunications, such as routing and load balancing. Many authors discussed relations between ant colony optimization algorithms and other approximate methods for optimization [10-13].

Recently, Rama Rao et al. [14] discussed ant colony optimization (ACO), which has been applied for inverse estimation of kinetic and film thickness parameters of biofilm. However, to the best of our knowledge, to date, no analytical expression corresponding to the concentrations of substrate is derived. The aim of this study is to explore an analytical method to represent the biofilm kinetics. The intension is to estimate the reaction rates of biofilm. The purpose of this communication is to derive an approximate analytical expression for the concentration for all values of parameters using the Adomian decomposition method [15-19].

2. MATHEMATICAL FORMULATION OF THE PROBLEM

This phenomenon has turned out to be crucial for the understanding the process of water purification in biofilm reactors. The schematic of the conceptual biofilm model showing the substrate concentrate profile in a segment of biofilm reactor is depicted in Fig. 1. A fixed bed biofilm reactor is setup to conduct experiments and to generate data. The schematic of experimental setup is shown in Fig. 2.

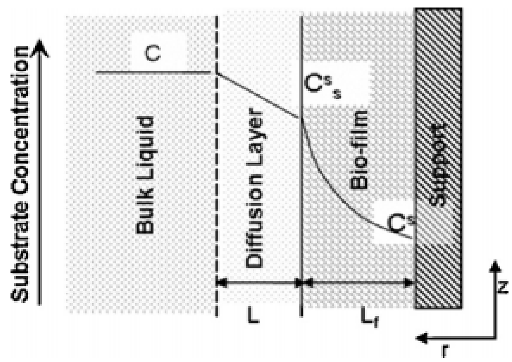

Fig. 1. Schematic of conceptual biofilm model showing the substrate concentrate profile [14]

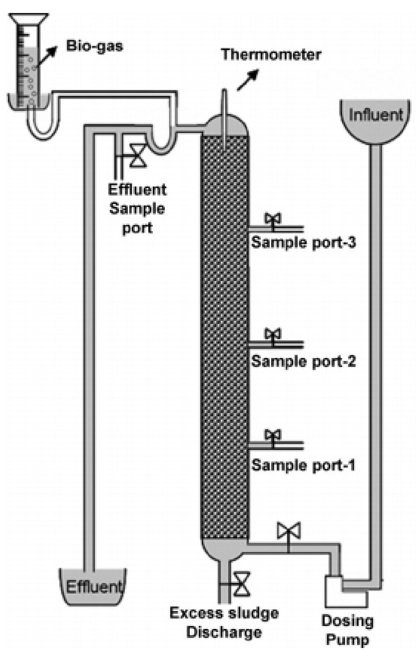

Fig. 2. Experimental setup [14]

2.1. Monod kinetics

The solid phase equation for the biofilm with c_s indicating concentration of the substrate is described by the following equation [14]:

$$\frac{D_f}{\xi^{a-1}}\frac{d}{d\xi}\left(\xi^{a-1}\frac{dc_s}{d\xi}\right)-\left(\frac{\mu_{\max}\rho_s}{Y}\right)\frac{c_s}{K_s+c_s}=0 \qquad (1)$$

Where $'a'$ is 1, 2, and 3 for planar, cylindrical and spherical geometries respectively. $D_f\ (m^2/day)$ and $K_S(kg/m^3)$ present substrate diffusion coefficient in the biofilm and half velocity constant respectively. The boundary conditions for the above equations are given by

$$\text{At } \xi=0,\quad \frac{dc_S}{d\xi}=0$$
$$\text{At } \xi=L_f,\quad k_g\left(c_S^S-c\right)=-D_f\frac{dc_S}{d\xi} \qquad (2)$$

Where $\xi, L_f, k_g, c_s, c_s^s$ and c represent the space coordinate in the biofilm (m), thickness of the biofilm (m), mass transfer coefficient (m/s), substrate concentration in the biofilm (kg/m^3), substrate concentration on the biofilm surface (kg/m^3) respectively. We introduce the following sets of dimensionless variables:

$$\mathrm{x}=\frac{\xi}{\mathrm{L_f}},\ \mathrm{s}=\frac{\mathrm{c_s}}{\mathrm{c_s}^{\mathrm{s}}},\ \gamma_{\mathrm{E}}=\frac{\mathrm{L_f}^2\mu_{\max}\rho_{\mathrm{s}}}{\mathrm{YD_fK_s}},\ \alpha=\frac{\mathrm{c_s}^{\mathrm{s}}}{\mathrm{K_s}},\ \mathrm{m}=\frac{\mathrm{k_g}}{\mathrm{L_fD_f}} \qquad (3)$$

Where $\mu_{\max}$, ρ_s and Y represents the maximum specific growth rate (day^{-1}), density of biomass (kg/m^3) and yield coefficient respectively. Then the nonlinear reaction-diffusion Eqns. (1-3) are expressed in the following non-dimensional form as follows:

$$\frac{d^2s}{dx^2}+\frac{a-1}{x}\frac{ds}{dx}=\frac{\gamma_E s}{1+\alpha s} \qquad (4)$$

With the boundary conditions

$$\text{At } x=0,\ \frac{ds}{dx}=0 \qquad (5)$$
$$\text{At } x=1,\ \frac{ds}{dx}=m(1-s)$$

2.1.1. Analytical expression of concentration of substrate for Monodkinetic models

A few nonlinear differential equations have known exact solution, but many which are important in applications do not. Sometimes these equations may be linearized by an expansion process in which nonlinear terms are discarded. When nonlinear terms make vital contributions to the solution this cannot be done, but sometimes it is enough to retain a few "small" ones. Then iteration method may be used to obtain in the solution. Recently, Sunil kumar and coworkers solve the nonlinear partial differential equations using new analytical method [21-22]. Many authors have applied the ADM to various problems and demonstrated the efficiency of the ADM for handling non-linear and solving various physics and engineering problems [15-19]. The normalized concentrations of the substrate are presented in Eqns. (4-5) defines boundary value problem [14]. The modified Adomian decomposition method [16] is used to give the approximate solutions of the non-linear Eqns. (4) and (5). The dimensionless reaction diffusion parameters are related to one another. Using ADM (refer Appendix A and B), we can obtain the following solutions to the Eqns. (4) and (5). For the case of planar geometry (*a=1*), the concentration of the substrate becomes

$$s(x)=1+\frac{\gamma_E}{1+\alpha}\left(\frac{x^2}{2}-\frac{1}{2}-\frac{1}{m}\right)+\frac{\gamma_E{}^2}{(1+\alpha)^3}\left(\frac{x^4}{24}-\frac{x^2(m+2)}{4m}+\frac{5m+6}{6m^2}+\frac{5}{24}\right) \tag{6}$$

For the case of cylindrical geometry $(a=2)$, the concentration of the substrate becomes

$$s(x)=1+\frac{\gamma_E}{1+\alpha}\left(\frac{x^2}{4}-\frac{1}{4}-\frac{1}{2m}\right)+\frac{\gamma_E{}^2}{(1+\alpha)^3}\left(\frac{x^4}{64}-\frac{x^2(m+2)}{16m}+\frac{3m+4}{16m^2}+\frac{3}{64}\right) \tag{7}$$

For the case of spherical geometry $(a=3)$, the concentration of the substrate becomes

$$s(x)=1+\frac{\gamma_E}{1+\alpha}\left(\frac{x^2}{6}-\frac{1}{6}-\frac{1}{3m}\right)+\frac{\gamma_E{}^2}{(1+\alpha)^3}\left(\frac{x^4}{120}-\frac{x^2(m+2)}{36m}+\frac{7m+10}{90m^2}+\frac{7}{360}\right) \tag{8}$$

2.2. Haldane kinetics

The solid phase equation for the biofilm with c_s the indicating concentration of the substrate is described by the following equation [14]:

$$\frac{D_f}{\xi^{a-1}}\frac{d}{d\xi}\left(\xi^{a-1}\frac{dc_s}{d\xi}\right)-\left(\frac{\mu_{\max}\rho_s}{Y}\right)\frac{c_s}{K_s+c_s+K_I c_s{}^2}=0 \tag{9}$$

where D_f and K_I is a substrate diffusion coefficient and substrate inhibition constant in the biofilm. The boundary conditions for this equation are given by

$$\begin{aligned} &At\ \xi=0,\quad \frac{dc_s}{d\xi}=0 \\ &At\ \xi=L_f,\quad k_g\left(c_s^s-c\right)=-D_f\frac{dc_s}{d\xi} \end{aligned} \tag{10}$$

We introduce the following set of dimensionless variables:

$$x=\frac{\xi}{L_f},\ s=\frac{c_s}{c_s^{\,s}},\ \gamma_E=\frac{L_f^{\,2}\mu_{\max}\rho_s}{YD_fK_s},\ \alpha=\frac{c_s^{\,s}}{K_s},\ m=\frac{k_g}{L_fD_f},\ \beta=\frac{K_I(c_s^{\,s})^2}{K_s} \tag{11}$$

Then the nonlinear reaction-diffusion Eqns. (1-3) are expressed in the following non-dimensional form as:

$$\frac{d^2s}{dx^2}+\frac{a-1}{x}\frac{ds}{dx}=\frac{\gamma_E s}{1+\alpha s+\beta s^2} \tag{12}$$

With the boundary conditions:

$$x=0,\ \frac{ds}{dx}=0$$
$$x=1,\ \frac{ds}{dx}=m(1-s) \tag{13}$$

2.2.1. Analytical expression of concentration of substrate for Haldane kinetic models

The normalized concentration of the substrates is presented in Eqns. (12-13). It defines the boundary value problem [14]. The modified Adomian decomposition method [16] is used to give the approximate solutions of the non-linear Eqns. (12) and (13) as follows:

For the case of planar geometry $(a=1)$, the concentration of the substrate becomes

$$s(x)=1+\frac{\gamma_E}{1+\alpha+\beta}\left(\frac{x^2}{2}-\frac{1}{2}-\frac{1}{m}\right)+\frac{(1-\beta)\gamma_E^{\,2}}{(1+\alpha+\beta)^3}\left(\frac{x^4}{24}-\frac{x^2(m+2)}{4m}+\frac{5m+6}{6m^2}+\frac{5}{24}\right) \tag{14}$$

For the case of cylindrical geometry $(a=2)$, the concentration of the substrate becomes

$$s(x)=1+\frac{\gamma_E}{1+\alpha+\beta}\left(\frac{x^2}{4}-\frac{1}{4}-\frac{1}{2m}\right)+\frac{(1-\beta)\gamma_E^{\,2}}{(1+\alpha+\beta)^3}\left(\frac{x^4}{64}-\frac{x^2(m+2)}{16m}+\frac{3m+4}{16m^2}+\frac{3}{64}\right) \tag{15}$$

For the case of spherical geometry $(a=3)$, the concentration of the substrate becomes

$$s(x)=1+\frac{\gamma_E}{1+\alpha+\beta}\left(\frac{x^2}{6}-\frac{1}{6}-\frac{1}{3m}\right)+\frac{(1-\beta)\gamma_E^{\,2}}{(1+\alpha+\beta)^3}\left(\frac{x^4}{120}-\frac{x^2(m+2)}{36m}+\frac{7m+10}{90m^2}+\frac{7}{360}\right) \tag{16}$$

2.3. Numerical Simulation

The non-linear differential equations (4) and (12) for the given initial–boundary conditions are being solved numerically. The function pdex, in MATLAB software which is a function of solving the initial–boundary value problems for non-linear ordinary differential equations is used to solve this equation. Its numerical solution is compared with analytical results using modified Adomian decomposition method and it gives a satisfactory result. The MATLAB program is also given in Appendix C.

3. DISCUSSION

Equations (6)-(8) and (14)-(16) are the new and simple analytical expressions of concentrations of substrate for Monod and Haldane kinetics respectively. Concentration of substrate versus distance is plotted in Fig. 3-8. The parameter γ_E describes the relative importance of diffusion and thickness of the biofilm.

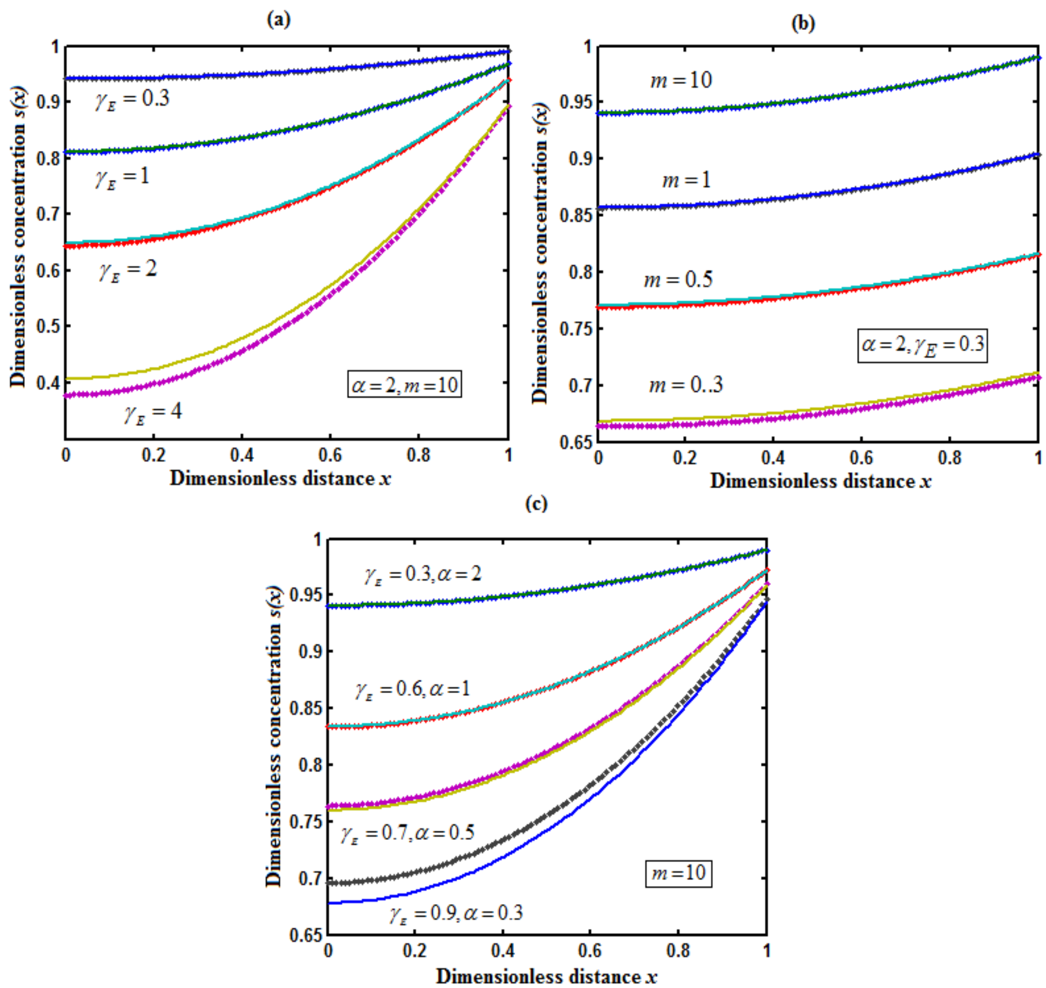

Fig. 3. Dimensionless concentration of $s(x)$ versus dimensionless distance x is plotted using Eqn. (6). (a) For various values of the parameter γ_E and for some fixed values of parameters $m=10, \alpha=2$. (b) For various values of the parameter m and for some fixed values of parameters $\gamma_E=0.3, \alpha=2$. (c) For various values of parameters α, γ_E and for some fixed values of the parameters $m=10$

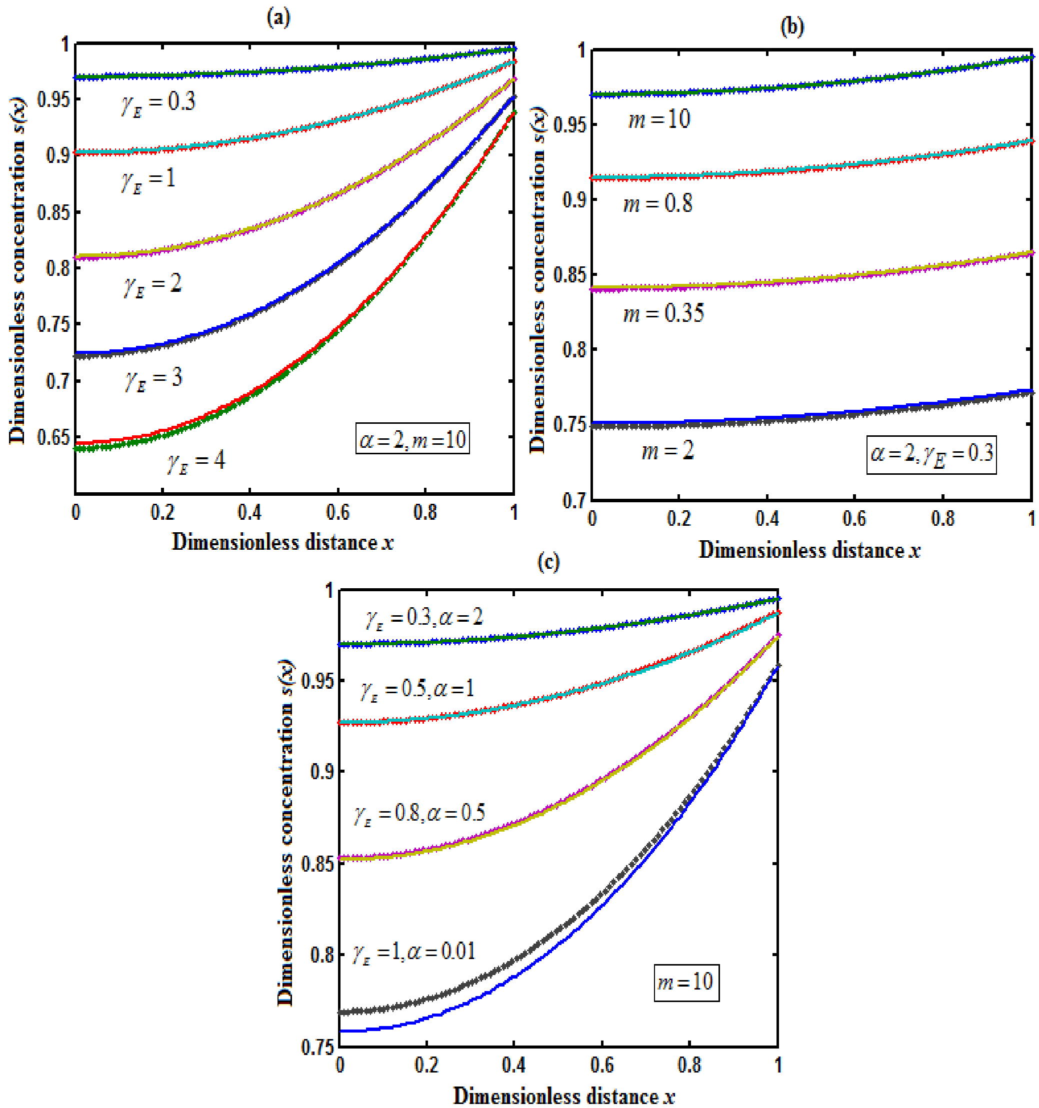

Fig. 4. Dimensionless concentration of $s(x)$ versus dimensionless distance x is plotted using Eqn. (7). (a) For various values of the parameter γ_E and for some fixed values of the parameters $m = 10, \alpha = 2$. (b) For various values of the parameter m and for some fixed values of the parameters $\gamma_E = 0.3, \alpha = 2$. (c) For various values of the parameters α, γ_E and for some fixed values of the parameter $m = 10$

Figs.3 (a)–8 (a) describe that the concentration rises slowly when $x \geq 0.4$ and rises abruptly when $x \geq 0.4$ for all values of diffusion coefficient γ_E. The concentration increases along with the value of m for some fixed diffusion coefficient γ_E and half velocity α in Figs. 3 (b) -8 (b). The concentration is uniform for all values of x and for some fixed value of other parameters. From Figs.3 (c) – 8 (c), it is evident that the concentration decreases when the diffusion coefficient increases and the half velocity constant α decreases. Figs. 6 (d) – 8 (d) is inferred that the parameter γ_E increases (diffusion coefficient decreases) or and substrate inhibition constant β decreases simultaneously when the concentration decreases for a fixed value of half velocity α and m.

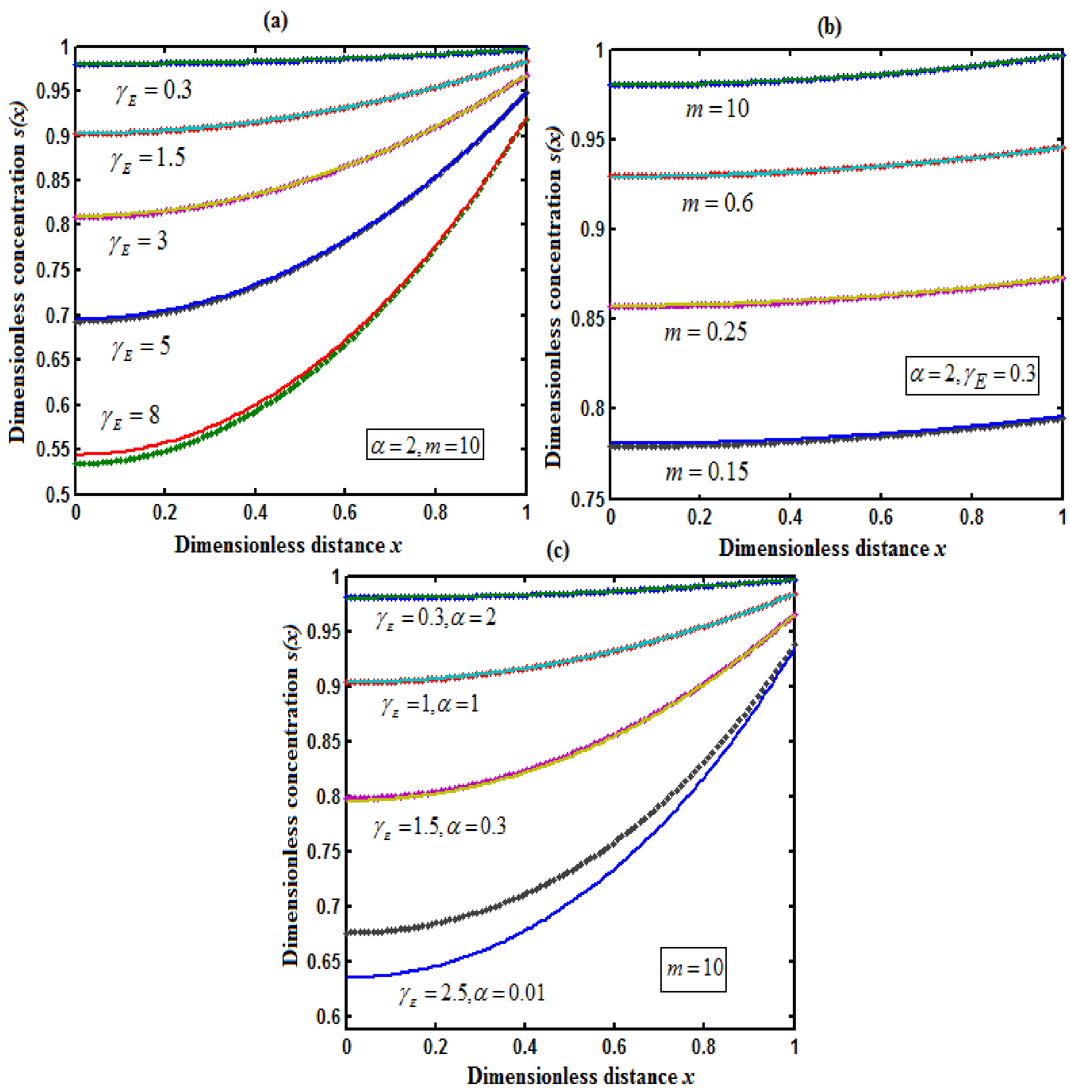

Fig. 5. Dimensionless concentration of $s(x)$ versus dimensionless distance x is plotted using Eqn. (8). (a) For various values of the parameter γ_E and for some fixed values of the parameters $m=10, \alpha=2$. (b) For various values of the parameter m and for some fixed values of the parameters $\gamma_E=0.3, \alpha=2$. (c) For various values of the parameters α, γ_E and for some fixed values of the parameter $m=10$

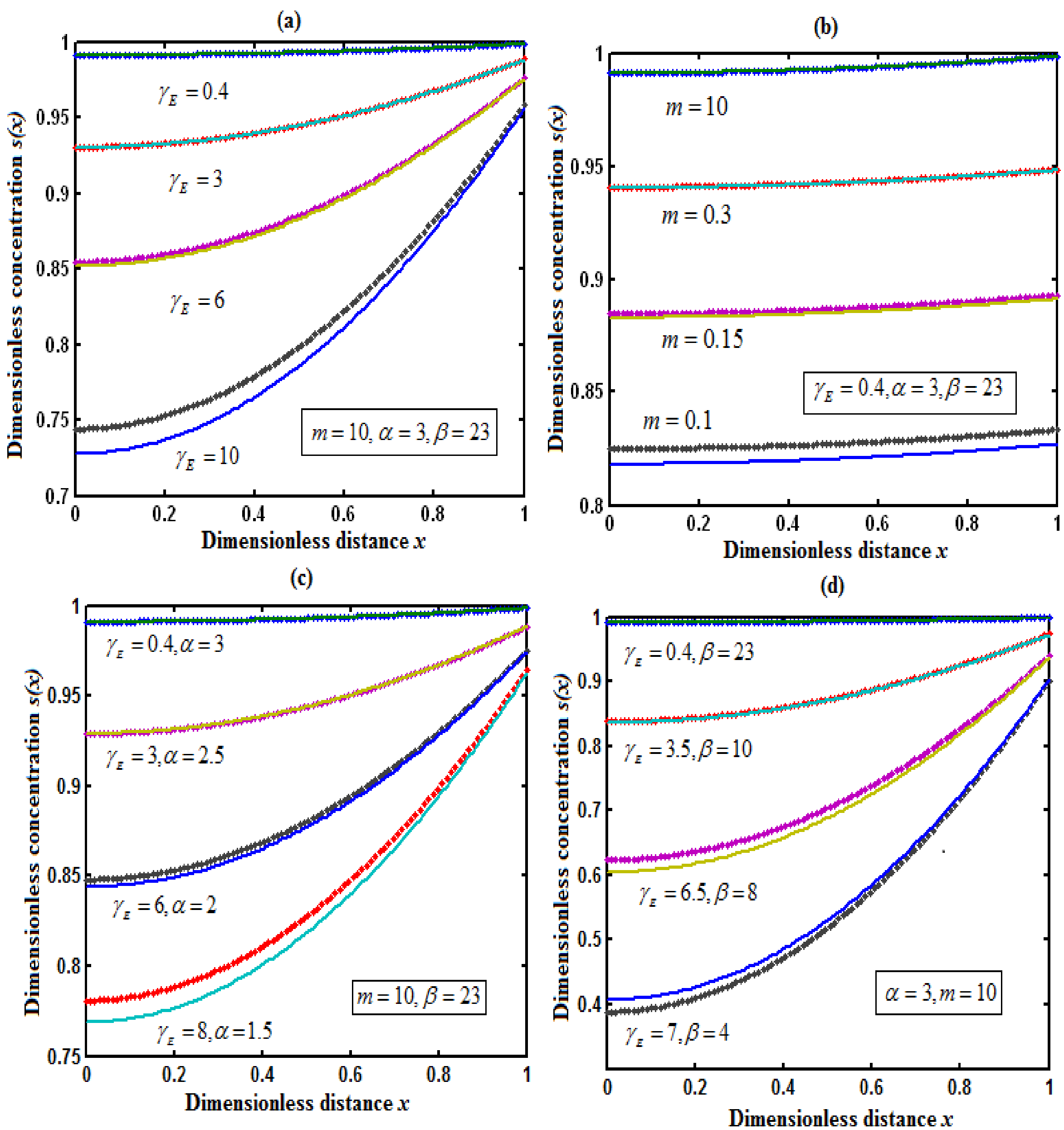

Fig. 6. Dimensionless concentration of $s(x)$ versus dimensionless distance x is plotted using Eqn. (14). (a) For various values of the parameter γ_E and for some fixed values of the parameters $m = 10, \alpha = 3$ and $\beta = 23$. (b) For various values of the parameter m and for some fixed values of the parameters $\gamma_E = 0.4$, $\alpha = 3$ and $\beta = 23$. (c) For various values of the parameters γ_E, α and for some fixed values of the parameters $m = 10, \beta = 23$. (d) For various values of the parameters γ_E, β and for some fixed values of the parameters $\alpha = 3$ and $m = 10$

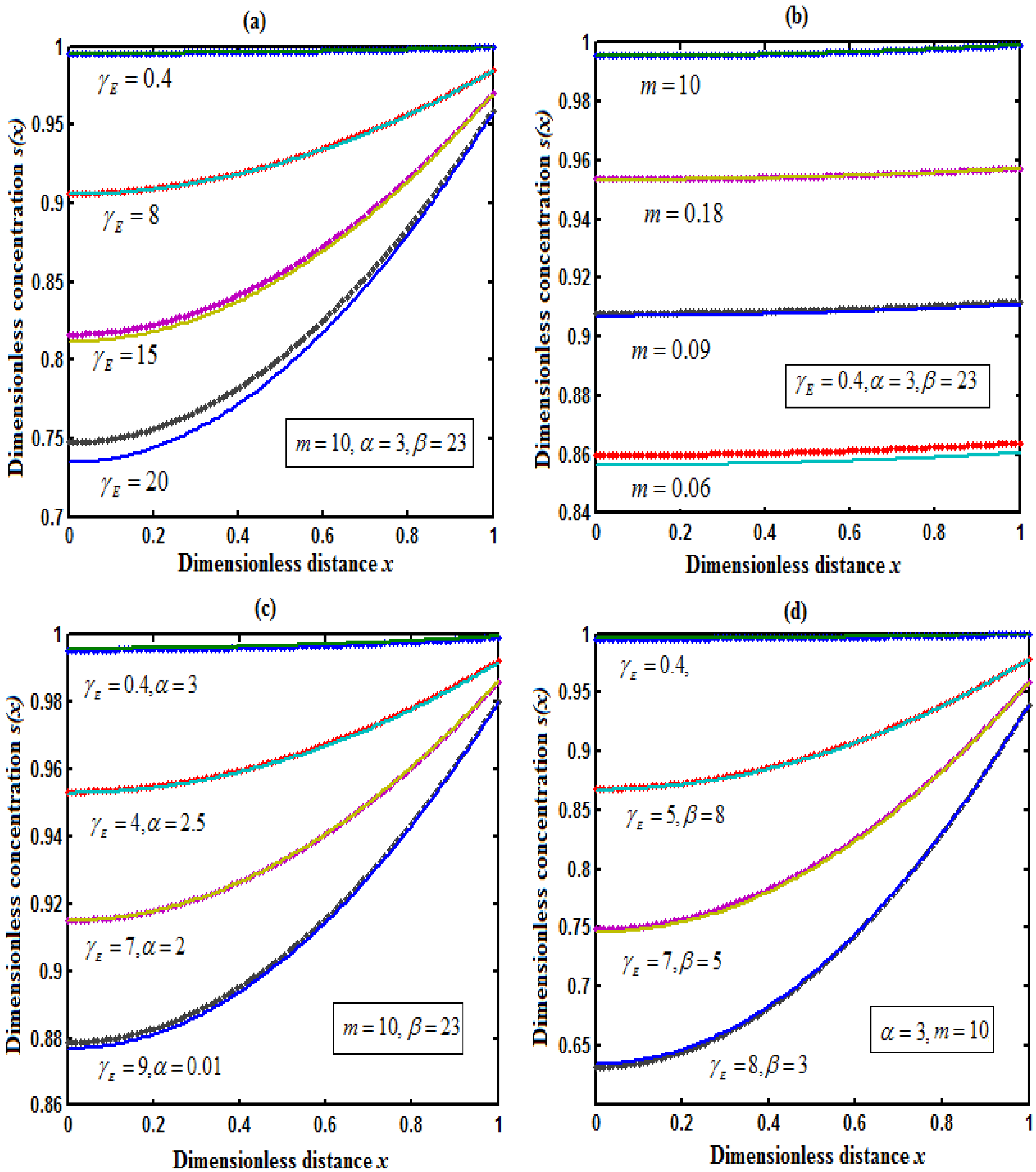

Fig. 7. Dimensionless concentration of $s(x)$ versus dimensionless distance x is plotted using Eqn. (15). (a) For various values of the parameter γ_E and for some fixed values of the parameters $m = 10, \alpha = 3$ and $\beta = 23$. (b) For various values of the parameter m and for some fixed values of the parameters $\gamma_E = 0.4$, $\alpha = 3$ and $\beta = 23$. (c) For various values of the parameters γ_E, α and for some fixed values of the parameters $m = 10, \beta = 23$. (d) For various values of the parameters γ_E, β and for some fixed values of the parameters $\alpha = 3$ and $m = 10$

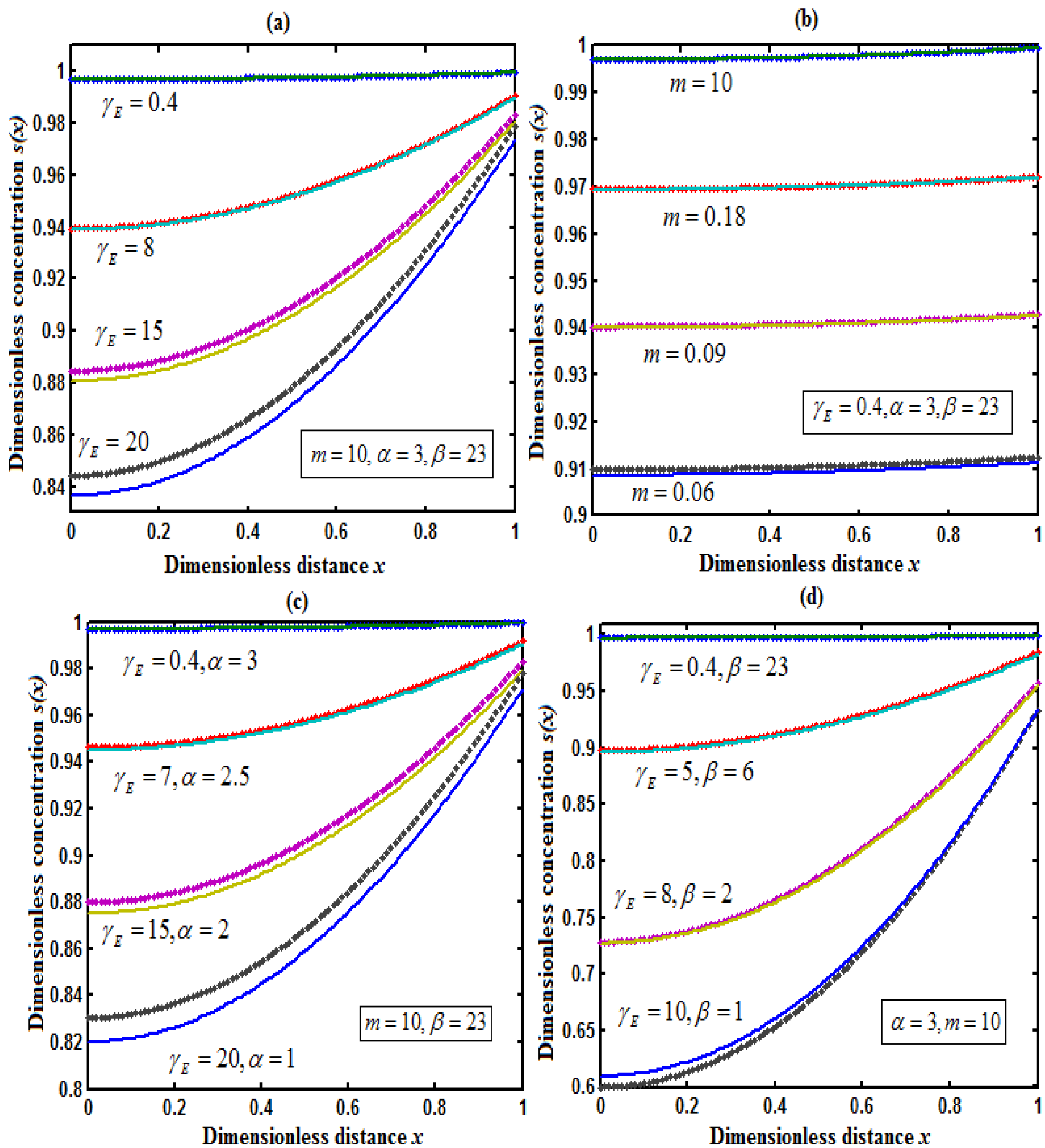

Fig. 8. Dimensionless concentration of $s(x)$ versus dimensionless distance x is plotted using Eqn. (16). (a) For various values of the parameter γ_E and for some fixed values of the parameters $m=10, \alpha=3$ and $\beta=23$. (b) For various values of the parameter m and for some fixed values of the parameters $\gamma_E=0.4$, $\alpha=3$ and $\beta=23$. (c) For various values of the parameters γ_E, α and for some fixed values of the parameters $m=10, \beta=23$. (d) For various values of the parameters γ_E, β and for some fixed values of the parameters $\alpha=3$ and $m=10$

4. CONCLUSION

We have presented a mathematical modeling of biofilm reactor. A nonlinear time independent partial differential equations in fixed bed biofilm reactor has been solved analytically using modified Adomian decomposition method. The analytical expressions of substrate concentration $s(x)$ for planar, cylinder and spherical for different values of parameters have been reported. This is an extremely simple method and it is also a promising method to solve the nonlinear equations. The analytical result is very useful to optimize the film thickness, half velocity constant and substrate inhibition constant. This method can be easily extended to solve the all kinds of non-linear equations in biofilm reactor, biofuel cell, biosensors and biofilm kinetics and so on.

Acknowledgements

This work was supported by the Department of Science and Technology (DST) (No. SB/S1/PC-50/2012). The authors are thankful to The Principal, The Madura College, Madurai and The Secretary, Madura College Board, Madurai for their encouragement.

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Appendix A: Basic concept of modified Adomian decomposition method

Consider the singular boundary value problem of $n+1$ order nonlinear differential equation in the form

$$y^{(n+1)}+\frac{m}{x}y^{(n)}+N\,y=g(x),$$
$$y(0)=a_0,\,y'(0)=a_1,\ldots,y^{n-1}(0)=a_{n-1},\;y(b)=c \tag{A.1}$$

where N is a non-linear differential operator of order less than n, $g(x)$ is given function and $a_0,a_1,\ldots a_{n-1},c,b$ are given constants. We propose the new differential operator, as below

$$L=x^{-1}\frac{d^n}{dx^n}x^{1+n-m}\frac{d}{dx}x^{m-n}(.) \tag{A.2}$$

where $m\le n, n\ge 1$, so, the problem can be written as

$$L^{-1}(.)=g(x)-Ny \tag{A.3}$$

The inverse operator L^{-1} is therefore considered a $n+1$ fold integral operator, as below [15-16]

$$L^{-1}(.)=x^{n-m}\int_b^x x^{m-n-1}\int_0^x\int_0^x\ldots\int_0^x x(.)dx\ldots dx. \tag{A.4}$$

By applying L^{-1} on (A.3), we have

$$y(x)=\phi(x)+L^{-1}g(x)L^{-1}Ny \tag{A.5}$$

Such that

$$L\phi(x)=0$$

The Adomian decomposition method introduce the solution $y(x)$ and the nonlinear function Ny by infinite series

$$y(x)=\sum_{n=0}^{\infty}y_n(x) \tag{A.6}$$

and

$$Ny=\sum_{n=0}^{\infty}A_n \tag{A.7}$$

where the components $y_n(x)$ of the solution $y(x)$ will be determined recurrently. Specific algorithms were seen in [15-16] to formulate Adomian polynomials. The following algorithm:

$$A_0 = F(u),$$
$$A_1 = F(u_0)u_1,$$
$$A_2 = F(u_0)u_2 + \frac{1}{2}F^{''}(u_0)u_1^2, \tag{A.8}$$

$$A_3 = F(u_0)u_3 + \frac{1}{2}F^{''}(u_0)u_1u_2 + \frac{1}{3!}F^{'''}(u_0)u_1^{\ 3},$$

. . .

can be used constant Adomian polynomials, when $F(u)$ is a nonlinear function. By substituting (A.6) and (A.7) in to (A.5)

$$\sum_{n=0}^{\infty} y_n = \phi(x) + L^{-1}g(x) - L^{-1}\sum_{n=0}^{\infty} A_n \tag{A.9}$$

Through using modified Adomian decomposition method, the components $y_n(x)$ can be determined as

$$\begin{aligned} y_0(x) &= A + L^{-1}g(x) \\ y_{n+1}(x) &= -L^{-1}(A_n), n \geq 0 \end{aligned} \tag{A.10}$$

Which gives

$$\begin{aligned} y_0(x) &= A + L^{-1}g(x) \\ y_1(x) &= -L^{-1}(A_0) \\ y_2(x) &= -L^{-1}(A_1) \\ y_3(x) &= -L^{-1}(A_2) \\ &\ldots \end{aligned} \tag{A.11}$$

From (A.8) and (A.11), we can determine the components $y_n(x)$, and hence the series solution of $y(x)$ in (A.6) can be immediately obtained. For numerical purposes, the n- term approximate

$$\psi_n = \sum_{n=0}^{n-1} y_k \tag{A.12}$$

can be used to approximate the exact solution. The approach presented above can be validated by testing it on a variety of several linear and nonlinear initial value problems.

Appendix B

In this appendix, we indicate how Eq. (6) in this paper derived. Furthermore, an ADM was constructed to determine the solution of Eq. (4) for $(a=1)$ in the operator form,

$$Ls = \frac{\gamma_E\, s}{1+\alpha\, s} \tag{B.1}$$

where $L = \dfrac{d^2}{dx^2}$, Applying the inverse operator L^{-1} on both sides of Eq. (B.1) yields

$$s(x) = Ax + B + \frac{\gamma_E\, s}{1+\alpha\, s} \tag{B.2}$$

Where A and B are the constants of integration. We let,

$$s(x) = \sum_{n=0}^{\infty} s_n \tag{B.3}$$

$$N[s(x)] = \sum_{n=0}^{\infty} A_n \tag{B.4}$$

Where $N[s(x)] = \dfrac{\gamma_E\, s}{1+\alpha\, s}$ (B.5)

From the eqns (B.3), (B.4) and (B.5), Eq. (B.2) gives

$$\sum_{n=0}^{\infty} s_n(x) = Ax + B + \frac{\gamma_E\, s}{1+\alpha\, s} \tag{B.6}$$

We identify the zeroth component as

$$s_0(x) = Ax + B \tag{B.7}$$

And the remaining components as the recurrence relation

$$s_{n+1} = \gamma_E\, L^{-1} A_n \,; n \geq 0 \tag{B.8}$$

where A_n are the Adomian polynomials of $s_{0,}\, s_{1,} \ldots, s_n$ .We can find the first few A_n as follows:

$$s_0(x) = 1 \tag{B.9}$$

$$s_1(x)=\frac{\gamma_E}{1+\alpha}\left(\frac{x^2}{2}-\frac{1}{2}-\frac{1}{m}\right) \tag{B.10}$$

$$s_2(x)=\frac{\gamma_E{}^2}{(1+\alpha)^3}\left(\frac{x^4}{24}-\frac{x^2(m+2)}{4m}+\frac{5m+6}{6m^2}+\frac{5}{24}\right) \tag{B.11}$$

Adding (B.9), (B .10) and (B.11), we get the concentration substrate Eq. (4) for $a=1$ (planar). Similarly, to obtained the concentration of substrate by solving Eqns. (4) and (12) for $a=2$ (cylindrical) and $a=3$ (spherical).

Appendix C

Scilab/Matlab program for the numerical solution of equation (4).

```
function pdex4
m = 0;
x = linspace(0,1);
t = linspace(0,100000);
sol = pdepe(m,@pdex4pde,@pdex4ic,@pdex4bc,x,t);
u1 = sol(:,:,1);
%————————————————————————————
Figure
plot(x,u1(end,:))
title('u1(x,t)')
xlabel('Distance x')
ylabel('u1(x,1)')
function [c,f,s] = pdex4pde(x,t,u,DuDx)
c =1;
f =1.* DuDx;
e=0.3;alpha=2;
F =-(e*u(1))/((1+(alpha*u(1))));
s =F;
% ————————————————————————————
function u0 = pdex4ic(x);
u0 = [0];
% ————————————————————————————
function [pl,ql,pr,qr] = pdex4bc(xl,ul,xr,ur,t)
j=10;
pl = [0];
ql = [1];
pr = [-j*(1-ur(1))];
qr = [1];
```

Appendix D

Nomenclature

Symbols	Définitions	Units
a	Constant (=1, 2, and 3 for planar, cylindrical and spherical pellets)	-
c_s	Substrate concentration in the biofilm	kg/m^3
c_s^s	Substrate concentration on the biofilm surface	kg/m^3
D_f	Substrate diffusion coefficient in the biofilm	m^2/day
k_g	Mass transfer coefficient	m/s
K_I	Substrate inhibition constant	m^3/day
K_s	Half velocity constant	kg/m^3
L_f	Thickness of the biofilm	m
m	Dimensionless parameter	none
s	Dimensionless substrate concentration in the biofilm	none
x	Dimensionless spaces coordinate in the biofilm	none
Y	Yield coefficient	none

Greek symbols

α	Dimensionless half velocity constant	-
β	Dimensionless substrate inhibition constant	-
γ_E	Dimensionless diffusion coefficient	-
$\mu_{\max}$	Maximum specific growth rate	day^{-1}
ρ_s	Density of biomass	kg/m^3
ξ	Spaces coordinate in the biofilm	m